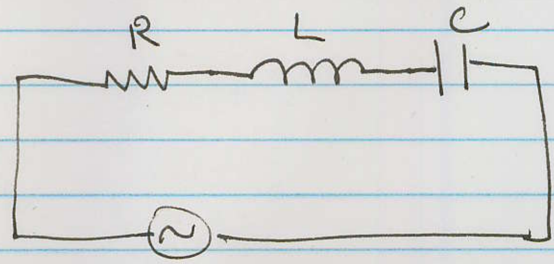
The RLC Series Circuit



$V = V_p \sin\omega t$

Objective : 1. To determine the phase angle "ϕ" between the current and applied voltage.

2. Determine "I_p" where
$$I = I_p \sin(\omega t + \phi)$$

- First, note that since the elements are in series, the current everywhere in the circuit must be the same at any instant.

- That is, the ac current at all points in a series ac circuit has the same amplitude and phase.

As we found out earlier, the voltage across each element will have different amplitudes and phases.

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$$V_R = I_p R \sin\omega t = V_{Rp} \sin\omega t$$

$$V_L = I_p X_L \sin\left(\omega t + \frac{\pi}{2}\right) = V_{Lp} \cos\omega t$$

$$V_C = I_p X_C \sin\left(\omega t - \frac{\pi}{2}\right) = -V_{Cp} \cos\omega t$$

X_C = capacitive reactance

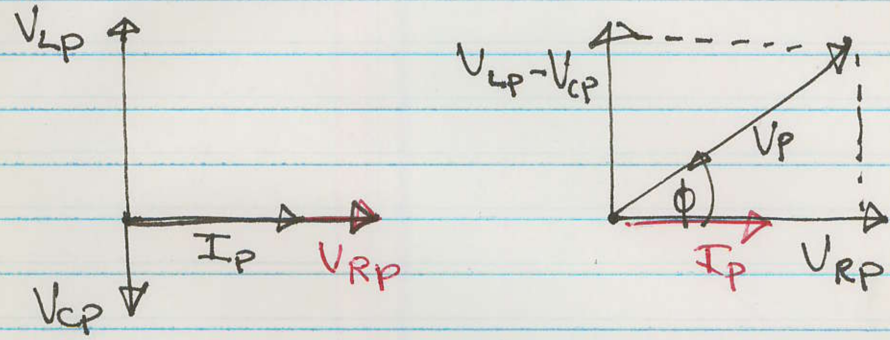
X_L = inductive reactance

Where V_{Rp}, V_{Lp}, and V_{Cp} are the peak voltages across each element.

$$V_{Rp} = I_p R \qquad X_L = \omega L$$

$$V_{Lp} = I_p X_L \qquad X_C = \frac{1}{\omega C}$$

$$V_{Cp} = I_p X_C$$



$X_L > X_C$

$$V_p = \sqrt{V_{Rp}^2 + (V_{Lp} - V_{Cp})^2}$$

$$V_p = \sqrt{(I_p R)^2 + (I_p X_L - I_p X_C)^2}$$

$$\boxed{V_p = I_p \sqrt{R^2 + (X_L - X_C)^2}}$$

$$X_L = \omega L$$

$$X_C = \frac{1}{\omega C}$$

$$I_p = \frac{V_p}{\sqrt{R^2 + (X_L - X_C)^2}}$$

We define the impedance of the circuit by the following expression:

$$\boxed{Z \equiv \sqrt{R^2 + (X_L - X_C)^2}} \quad \text{(Impedance)}$$

$$\boxed{V_P = I_P Z}$$

From the phasor diagram:

$$\tan\phi = \frac{V_{LP} - V_{CP}}{V_{RP}} = \frac{I_P X_L - I_P X_C}{I_P R}$$

$$\tan\phi = \frac{X_L - X_C}{R}$$

$$\boxed{\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)}$$

"ϕ" is the phase angle between the current and voltage.

For example:

when $X_L > X_C$ then $\phi > 0$
$\Rightarrow$ the current lags behind the applied voltage

When $X_L < X_C$ then $\phi < 0$
$\Rightarrow$ the current leads the applied voltage

current is pure resistive

When $X_L = X_C$ then $\phi = 0$

$\Rightarrow$ current and applied voltage are in phase

$$I_p = \frac{V_p}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{V_p}{R}$$

The frequency at which this occurs is called the resonance frequency.

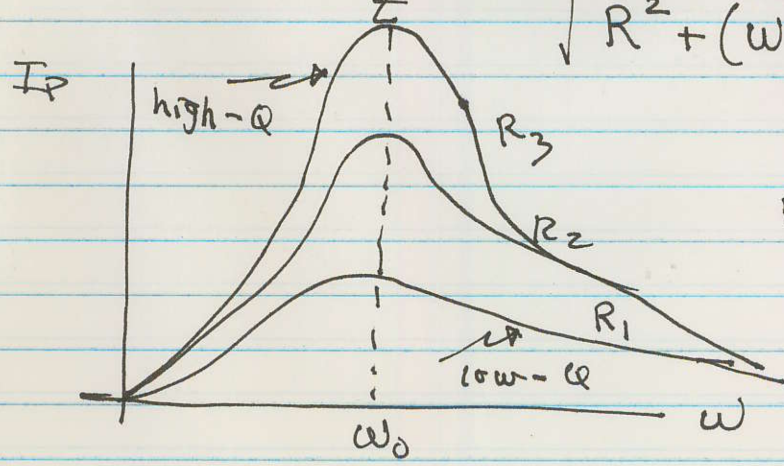
$$X_L = X_C$$

$$\omega_0 L = \frac{1}{\omega_0 C}$$

$$\boxed{\omega_0 = \frac{1}{\sqrt{LC}}}$$ Resonance Frequency

Graph of I_p:

$$I_p = \frac{V_p}{Z} = \frac{V_p}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$



$R_3 < R_2 < R_1$