Math 001B Assignment 9.4 Last Name______________________1st ______________

DIRECTIONS: To receive full credit, you must provide complete legible solutions to the following problems in the space provided.

1. The Pacific halibut fishery has been modeled by the differential equation

$$\frac{dy}{dt} = ky\left(1 - \frac{y}{M}\right)$$

where y(t) is the biomass (the total mass of the members of the population) in kilograms at time t (measured in years), the carrying capacity is estimated to be M = 7 × 107 kg, and k = 0.74 per year.

a. If y(0) = 2 × 107 kg, find the biomass a year later. Ans ______________

b. How long will it take for the biomass to reach 4 × 107 kg? Ans_______________

2.a Use the fact that the population was 250 million in 1990 (t = 0) to formulate a logistic model for the US population. (Assume the carrying capacity is 4000 million. Assume P is the population in millions, k is the relative growth rate, and t is the time in years since 1990.)

Ans_____________________

b. Determine the value of k in your model by using the fact that the population in 2000 was 275 million. Ans_____________________

c. Use your model to predict the US population in the year 2100. Ans_____________________

3. Let's modify the logistic differential equation of this example as follows:

$$\frac{dP}{dt} = 0.2P\left(1 - \frac{p}{1000}\right) - 32$$

a. Suppose P(t)represents a fish population at time t, where t is measured in weeks. Explain the meaning of the final term in the equation (–32).

Ans The term -32 represents a harvesting of fish at a constant rate — in this case, 32 fish/week This is the rate at which fish are caught.

b. Draw a direction field for this differential equation. Use the direction field to sketch several solution curves. Use a graphing utility to produce the direction field then paste it on the space below.

c. What are the equilibrium solutions? Ans____________________

d. Describe what happens to the fish population for various initial populations.

For $0 < P0 < 200$, Ans____________________

For $P_0 = 200$, Ans____________________

For $200 < P0 < 800$, Ans____________________

For $P_0 = 800$, P(t) Ans____________________

For $P_0 > 800$, P(t) Ans____________________

e. Solve this differential equation explicitly, either by using partial fractions or with a computer algebra system. Use the initial populations 150 .

Ans____