Section 6.6 and 6.7 with finance review questions are included in this document for your convenience for studying for quizzes and exams for Finance Calculations for Math 11.

Section 6.6 focuses on identifying the strategy for HOW to do the calculation.
Section 6.7 focuses on getting the answer to the problem, working it through completely.
Answers for 6.6 (Strategy - which formula to use) and 6.7 (numerical answers) are at the end of this document.

In section 6.6, you should be able to do questions 1-7, 11-20 for the quiz that covers 6.1 – 6.4.

In section 6.6, you should be able to do all the questions for the exam that covers 6.1 – 6.5

In section 6.7, you should be able to do questions 1-5, 7-8, 10, 11, 13, 16, 17, 19-34 for the quiz that covers 6.1 – 6.4

In section 6.7, you should be able to do questions 1-8, 10- 17, 19-35 for the exam that covers 6.1 – 6.5

You can NOT print out anything from this document to bring as notes to an exam or quiz.
You CAN handwrite anything you want from this document into your notes for an exam or quiz.

The three formulas we use most commonly involving compounding interest n times a year are

Compound Interest with a lump sum deposit: $A = P(1+r/n)^{nt}$

Future Accumulated Value (at the end) of an annuity: $A = m[(1+r/n)^{nt}-1]/(r/n)$

Present Value (at the beginning) of an annuity: $P(1+r/n)^{nt} = m[(1+r/n)^{nt}-1]/(r/n)$
Also Principal or Present value (at the beginning) of a LOAN.

We also have other formulas for a lump sum deposit relating A and P

simple interest $A = P(1+rt)$

continuous compounding $A = Pe^{rt}$

And finally we have effective rate formulas (APY or APR)

$r_{EFF} = (1+r/n)^n-1$ for compounding n times per year

$r_{EFF} = e^r-1$ for continuous compounding.

6.6 Classification of Finance Problems

In this section, you will review the concepts of chapter 6 to:

1. *re-examine the types of financial problems and classify them.*
2. *re-examine the vocabulary words used in describing financial calculations*

We'd like to remind the reader that the hardest part of solving a finance problem is determining the category it falls into. So in this section, we will emphasize the classification of problems rather than finding the actual solution.

We suggest that the student read each problem carefully and look for the word or words that may give clues to the kind of problem that is presented. For instance, students often fail to distinguish a lump-sum problem from an annuity. Since the payments are made each period, an annuity problem contains words such as each, every, per etc.. One should also be aware that in the case of a lump-sum, only a single deposit is made, while in an annuity numerous deposits are made at equal spaced time intervals. To help interpret the vocabulary used in the problems, we include a glossary at the end of this section.

Students often confuse the present value with the future value. For example, if a car costs \$15,000, then this is its present value. Surely, you cannot convince the dealer to accept \$15,000 in some future time, say, in five years. Recall how we found the installment payment for that car. We assumed that two people, Mr. Cash and Mr. Credit, were buying two identical cars both costing \$15, 000 each. To settle the argument that both people should pay exactly the same amount, we put Mr. Cash's cash of \$15,000 in the bank as a lump-sum and Mr. Credit's monthly payments of x dollars each as an annuity. Then we make sure that the future values of these two accounts are equal. As you remember, at an interest rate of 9%

the future value of Mr. Cash's lump-sum was $\$15,000(1 + .09/12)^{60}$, and

the future value of Mr. Credit's annuity was $\dfrac{x[(1+.09/12)^{60} - 1]}{.09/12}$.

To solve the problem, we set the two expressions equal and solve for m.

The present value of an annuity is found in exactly the same way. For example, suppose Mr. Credit is told that he can buy a particular car for \$311.38 a month for five years, and Mr. Cash wants to know how much he needs to pay. We are finding the present value of the annuity of \$311.38 per month, which is the same as finding the price of the car. This time our unknown quantity is the price of the car. Now suppose the price of the car is P, then

the future value of Mr. Cash's lump-sum is $P(1 + .09/12)^{60}$, and

the future value of Mr. Credit's annuity is $\dfrac{\$311.38[(1 + .09/12)^{60} - 1]}{.09/12}$.

Setting them equal we get,

$$P(1 + .09/12)^{60} = \frac{\$311.38[(1 + .09/12)^{60} - 1]}{.09/12}$$

$$P(1.5657) = (\$311.38)(75.4241)$$

$$P(1.5657) = \$23,485.57$$

$$P = \$15,000.04$$

CLASSIFICATION OF PROBLEMS AND EQUATIONS FOR SOLUTIONS

We now list six problems that form a basis for all finance problems. Further, we classify these problems and give an equation for the solution.

◆***Problem 1*** If \$2,000 is invested at 7% compounded quarterly, what will the final amount be in 5 years?

Classification: **Future (accumulated) Value of a Lump-sum** or FV of a lump-sum.

Equation: $FV = A = \$2000(1 + .07/4)^{20}$.

◆***Problem 2*** How much should be invested at 8% compounded yearly, for the final amount to be \$5,000 in five years?

Classification: **Present Value of a Lump-sum** or PV of a lump-sum.

Equation: $PV(1 + .08)^5 = \$5,000$

◆***Problem 3*** If \$200 is invested *each* month at 8.5% compounded monthly, what will the final amount be in 4 years?

Classification: **Future (accumulated) Value of an Annuity** or FV of an annuity.

Equation: $FV = A = \dfrac{\$200[(1 + .085/12)^{48} - 1]}{.085/12}$

◆***Problem 4*** How much should be invested *each* month at 9% for it to accumulate to \$8,000 in three years?

Classification: **Sinking Fund Payment**

Equation: $\dfrac{m[(1 + .09/12)^{36} - 1]}{.09/12} = \$8,000$

◆***Problem 5*** Keith has won a lottery paying him \$2,000 *per* month for the next 10 years. He'd rather have the entire sum now. If the interest rate is 7.6%, how much should he receive?

Classification: **Present Value of an Annuity** or PV of an annuity.

Equation: $PV(1 + .076/12)^{120} = \dfrac{\$2000[(1 + .076/12)^{120} - 1]}{.076/12}$

◆***Problem 6*** Mr. A has just donated \$25,000 to his alma mater. Mr. B would like to donate an equivalent amount, but would like to pay by monthly payments over a five year period. If the interest rate is 8.2%, determine the size of the monthly payment?

Classification: **Installment Payment**.

Equation: $\dfrac{m[(1 + .082/12)^{60} - 1]}{.082/12} = \$25,000(1 + .082/12)^{60}$.

GLOSSARY: VOCABULARY AND SYMBOLS USED IN FINANCIALCALCULATIONS

As we've seen in these examples, it's important to read the problems carefully to correctly identify the situation. It is essential to understand to vocabulary for financial problems. Many of the vocabulary words used are listed in the glossary below for easy reference.

t	Term	Time period for a loan or investment. In this book t is represented in years and should be converted into years when it is stated in months or other units.
P	Principal	Principal is the amount of money borrowed in a loan. If a sum of money is invested for a period of time, the sum invested at the start is the Principal.
P	Present Value	Value of money at the beginning of the time period.
A	Accumulated Value Future Value	Value of money at the end of the time period
D	Discount	In loans involving simple interest, a discount occurs if the interest is deducted from the loan amount at the beginning of the loan period, rather than being repaid at the end of the loan period.
m	Periodic Payment	The amount of a constant periodic payment that occurs at regular intervals during the time period under consideration (examples: periodic payments made to repay a loan, regular periodic payments into a bank account as savings, regular periodic payment to a retired person as an annuity,)
n	Number of payment periods and compounding periods per year	In this book, when we consider periodic payments, we will always have the compounding period be the same as the payment period. In general the compounding and payment periods do not have to be the same, but the calculations are more complicated if they are different. If the periods differ, formulas for the calculations can be found in finance textbooks or various online resources. Calculations can easily be done using technology such as an online financial calculator, or financial functions in a spreadsheet, or a financial pocket calculator.
nt	Number of periods	nt = (number of periods per year)×(number of years) nt gives the total number of payment and compounding periods In some situations we will calculate nt as the multiplication shown above. In other situations the problem may state nt, such as a problem describing an investment of 18 months duration compounded monthly. In this example: nt = 18 months and n = 12; then t = 1.5 years but t is not stated explicitly in the problem. The TI-84+ calculators built in TVM solver uses N = nt.
r	Annual interest rate Nominal rate	The stated annual interest rate. This is stated as a percent but converted to decimal form when using financial calculation formulas. If a bank account pays 3% interest compounded quarterly, then 3% is the nominal rate, and it is included in the financial formulas as r = 0.03
r/n	Interest rate per compounding period	If a bank account pays 3% interest compounded quarterly, then r/n = 0.03/4 = 0. 075, corresponding to a rate of 0.75% per quarter. Some Finite Math books use the symbol i to represent r/n

r_{EFF}	Effective Rate Effective Annual Interest Rate APY Annual Percentage Yield APR Annual Percentage Rate	The effective rate is the interest rate compounded annually that would give the same interest rate as the compounded rate stated for the investment. The effective rate provides a uniform way for investors or borrowers to compare different interest rates with different compounding periods.
I	Interest	Money paid by a borrower for the use of money borrowed as a loan. Money earned over time when depositing money into a savings account, certificate of deposit, or money market account. When a person deposits money in a bank account, the person depositing the funds is essentially temporarily lending the money to the bank and the bank pays interest to the depositor.
	Sinking Fund	A fund set up by making payments over a period of time into a savings or investment account in order to save to fund a future purchase. Businesses use sinking funds to save for a future purchase of equipment at the end of the savings period by making periodic installment payments into a sinking fund.
	Annuity	An annuity is a stream of periodic payments. In this book it refers to a stream of constant periodic payments made at the end of each compounding period for a specific amount of time. In common use the term annuity generally refers to a constant stream of periodic payments received by a person as retirement income, such as from a pension. Annuity payments in general may be made at the end of each payment period (ordinary annuity) or at the start of each period (annuity due). The compounding periods and payment periods do not need to be equal, but in this textbook we only consider situations when these periods are equal.
	Lump Sum	A single sum of money paid or deposited at one time, rather than being spread out over time. An example is lottery winnings if the recipient chooses to receive a single "lump sum" one-time payment, instead of periodic payments over a period of time or as. Use of the word lump sum indicates that this is a one time transaction and is not a stream of periodic payments.
	Loan	An amount of money that is borrowed with the understanding that the borrower needs to repay the loan to the lender in the future by the end of a period of time that is called the term of the loan. The repayment is most often accomplished through periodic payments until the loan has been completely repaid over the term of the loan. However there are also loans that can be repaid as a single sum at the end of the term of the loan, with interest paid either periodically over the term or in a lump sum at the end of the loan or as a discount at the start of the loan.

SECTION 6.6 PROBLEM SET: CLASSIFICATION OF FINANCE PROBLEMS

Let the letters A, B, C, D, E and F be represented as follows:

A = FV of a lump-sum	C = FV of an annuity	E = Installment payment
B = PV of a lump-sum	D = Sinking fund payment	F = PV of an annuity

Classify each by writing the appropriate letter in the box, and write an equation for solution.

1) What monthly deposits made to an account paying 9% will grow to $10,000 in 4 years?

2) An amount of $4000 is invested at 6% compounded daily. What will the final amount be in 5 years?

3) David has won a lottery paying him $10,000 per month for the next 20 years. He'd rather have the whole amount in one lump sum now. If the current interest rate is 7%, how much money can he hope to get?

4) Each month Linda deposits $250 in an account that pays 9%. How much money will she have in 4 years?

5) Find the monthly payment for a $15,000 car if the loan is amortized over 4 years at a rate of 10%.

6) What lump-sum deposited in an account paying 7% compounded daily will grow to $10,000 in 5 years?

7) What amount of quarterly payments will amount to $250,000 in 5 years at a rate of 8%?

8) The Chang family bought their house 25 years ago. They had their loan financed for 30 years at an interest rate of 11% resulting in a payment of $1350 a month. Find the balance of the loan.

9-10) A 10-year $1000 bond pays $35 every six months. If the current interest rate is 8%, in order to find the fair market value of the bond, we need to find the following.

9) The present value of $1000.

10) The present value of the $35 per six month payments.

SECTION 6.6 PROBLEM SET: CLASSIFICATION OF FINANCE PROBLEMS

A = FV of a lump-sum	C = FV of an annuity	E = Installment payment
B = PV of a lump-sum	D = Sinking fund payment	F = PV of an annuity

11) What lump-sum deposit made today is equal to 33 monthly deposits of $500 if the interest rate is 8%?

12) What monthly deposits made to an account paying 10% will accumulated to $10,000 in six years?

13) A department store charges a finance charge of 1.5% per month on the outstanding balance. If Ned charged $400 three months ago and has not paid his bill, how much does he owe?

14) What will the value of $300 monthly deposits be in 10 years if the account pays 12% compounded monthly?

15) What lump-sum deposited at 6% compounded daily will grow to $2000 in three years?

16) A company buys an apartment complex for $5,000,000 and amortizes the loan over 10 years. What is the yearly payment if the interest rate is 14%?

17) In 2002, a house in Rock City cost $300,000. Real estate in Rock City has been increasing in value at the annual rate of 5.3%.. Find the price of that house in 2016.

18) You determine that you can afford to pay $400 per month for a car. What is the maximum price you can pay for a car if the interest rate is 11% and you want to repay the loan in 4 years?

19) A business needs $350,000 in 5 years. How much lump-sum should be put aside in an account that pays 9% so that five years from now the company will have $350,000?

20) A person wishes to have $500,000 in a pension fund 20 years from now. How much should he deposit each month in an account paying 9% compounded monthly?

SECTION 6.7 PROBLEM SET: CHAPTER REVIEW

1) Manuel borrows $800 for 6 months at 18% simple interest. How much does he owe at the end of 6 months?

2) The population of a city is 65,000 and expects to grow at a rate of 2.3% per year for the next 10 years. What will the population of this city be in 10 years?

3) The Gill family is buying a $250,000 house with a 10% down payment. If the loan is financed over a 30 year period at an interest rate of 4.8%, what is the monthly payment?

4) Find the monthly payment for the house in the above problem if the loan was amortized over 15 years.

5) You look at your budget and decide that you can afford $250 per month for a car. What is the maximum amount you can afford to pay for the car if the interest rate is 8.6% and you want to finance the loan over 5 years?

6) Mr. Nakahama bought his house in the year 1998. He had his loan financed for 30 years at an interest rate of 6.2% resulting in a monthly payment of $1500. In 2015, 17 years later, he paid off the balance of the loan. How much did he pay?

7) Lisa buys a car for $16,500, and receives $2400 for her old car as a trade-in value. Find the monthly payment for the balance if the loan is amortized over 5 years at 8.5%.

8) A car is sold for $3000 cash down and $400 per month for the next 4 years. Find the cash value of the car today if the money is worth 8.3% compounded monthly.

9) An amount of $2300 is borrowed for 7 months at a simple interest rate of 16%. Find the discount and the proceeds.

10) Marcus has won a lottery paying him $5000 per month for the next 25 years. He'd rather have the whole amount in one lump sum today. If the current interest rate is 7.3%, how much money can he hope to get?

11) In the year 2000, an average house in Star City cost $250,000. If the average annual inflation rate for the past years has been about 4.7%, what was the price of that house in 2015?

12) Find the 'fair market' value of a ten-year $1000 bond which pays $30 every six months if the current interest rate is 7%. What if the current interest rate is 5%?

13) A Visa credit card company has a finance charge of 1.5% per month (18% per year) on the outstanding balance. John owed $3200 and has been delinquent for 5 months. How much total does he owe, now?

14) You want to purchase a home for $200,000 with a 30-year mortgage at 9.24% interest. Find a) the monthly payment and b) the balance owed after 20 years.

15) When Jose bought his car, he amortized his loan over 6 years at a rate of 9.2%, and his monthly payment came out to be $350 per month. He has been making these payments for the past 40 months and now wants to pay off the remaining balance. How much does he owe?

16) A lottery pays $10,000 per month for the next 20 years. If the interest rate is 7.8%, find both its present and future values.

SECTION 6.7 PROBLEM SET: CHAPTER REVIEW

17) A corporation estimates it will need $300,000 in 8 years to replace its existing machinery. How much should it deposit each quarter in a sinking fund earning 8.4% compounded quarterly to meet this obligation?

18) Our national debt in 1992 was about $4 trillion. If the annual interest rate was 7% then, what was the daily interest on the national debt?

19) A business must raise $400,000 in 10 years. What should be the size of the owners' monthly payments to a sinking fund paying 6.5% compounded monthly?

20) The population of a city of 80,000 is growing at a rate of 3.2% per year. What will the population be at the end of 10 years?

21) A sum of $5000 is deposited in a bank today. What will the final amount be in 20 months if the bank pays 9% and the interest is compounded monthly?

22) A manufacturing company buys a machine for $500 cash and $50 per month for the next 3 years. Find the cash value of the machine today if the money is worth 6.2% compounded monthly.

23) The United States paid about 4 cents an acre for the Louisiana Purchase in 1803. Suppose the value of this property grew at a rate of 5.5% annually. What would an acre be worth in the year 2000?

24) What amount should be invested per month at 9.1% compounded monthly so that it will become $5000 in 17 months?

25) A machine costs $8000 and has a life of 5 years. It can be leased for $160 per month for 5 years with a cash down payment of $750. The current interest rate is 8.3%. Is it cheaper to lease or to buy?

26) If inflation holds at 5.2% per year for 5 years, what will be the cost in 5 years of a car that costs $16,000 today? How much will you need to deposit each quarter in a sinking fund earning 8.7% per year to purchase the new car in 5 years?

27) City Bank pays an interest rate of 6%, while Western Bank pays 5.8% compounded continuously. Which one is a better deal?

28) Ali has inherited $20,000 and is planning to invest this amount at 7.9% interest. At the same time he wishes to make equal monthly withdrawals to use up the entire sum in 5 years. How much can he withdraw each month?

29) Jason has a choice of receiving $300 per month for the next 5 years or $500 per month for the next 3 years. Which one is worth more if the current interest rate is 7.7%?

30) If a bank pays 6.8% compounded continuously, how long will it take to double your money?

31) A mutual fund claims a growth rate of 8.3% per year. If $500 per month is invested, what will the final amount be in 15 years?

32) Mr. Vasquez has been given two choices for his compensation. He can have $20,000 cash plus $500 per month for 10 years, or he can receive $12,000 cash plus $1000 per month for 5 years. If the interest rate is 8%, which is the better offer?

SECTION 6.7 PROBLEM SET: CHAPTER REVIEW

33) How much should Mr. Shackley deposit in a trust account so that his daughter can withdraw $400 per month for 4 years if the interest rate is 8%?

34) Mr. Albers borrowed $425,000 from the bank for his new house at an interest rate of 4.7%. He will make equal monthly payments for the next 30 years. How much money will he end up paying the bank over the life of the loan, and how much is the interest?

35) Mr. Tong puts away $500 per month for 10 years in an account that earns 9.3%. After 10 years, he decides to withdraw $1,000 per month. If the interest rate stays the same, how long will it take Mr. Tong to deplete the account?

36) An amount of $5000 is borrowed for 15 months at an interest rate of 9%. Find the monthly payment and construct an amortization schedule showing the monthly payment, the monthly interest on the outstanding balance, the amount of payment contributing towards debt, and the outstanding debt.

Strategies for Solving Questions in Section 6.6 Problem Set

1) We know A = $10,000. We need to find m. Use formula $A = m[(1+r/n)^{nt} - 1]/(r/n)$

2) We know P = 4000. We need to find A. $A = P(1+r/n)^{nt}$

3) We know m = $10,000. We need to find P. Use formula $P(1+r/n)^{nt} = m[(1+r/n)^{nt} - 1]/(r/n)$

4) We know m = $250. We need to find A. Use formula $A = m[(1+r/n)^{nt} - 1]/(r/n)$

5) We know P = $15000. We need to find m. Use formula $P(1+r/n)^{nt} = m[(1+r/n)^{nt} - 1]/(r/n)$

6) We know A = $10,000. We need to find P Use formula $A = P(1+r/n)^{nt}$

7) We know A = $250,000. We need to find m. Use formula $A = m[(1+r/n)^{nt} - 1]/(r/n)$

8) This is from section 6.5. We know m = $1350. Find P= outstanding balance of a loan when t = 30-25=5 years remaining on the loan. Use t = 5 with formula $P(1+r/n)^{nt} = m[(1+r/n)^{nt} - 1]/(r/n)$.

9 & 10) This is from section 6.5 Present value of a bond

9) Find P when A = $1000 10) Find P when m = $35 **Then add both values for P together.**

9) Use $A = P(1+r/n)^{nt}$ 10) Use $P(1+r/n)^{nt} = m[(1+r/n)^{nt} - 1]/(r/n)$

11) We know m = $500. We need to find P. Use formula $P(1+r/n)^{nt} = m[(1+r/n)^{nt} - 1]/(r/n)$

12) We know A = $10,000. We need to find m. Use formula $A = m[(1+r/n)^{nt} - 1]/(r/n)$

13) We know P = $400. We need to find A. Use formula $A = P(1+r/n)^{nt}$

14) We know m = $300. We need to find A. Use formula $A = m[(1+r/n)^{nt} - 1]/(r/n)$

15) We know A = $2000. We need to find P. Use formula $A = P(1+r/n)^{nt}$

16) We know P= $5,000,000. We need to find m. Use formula $P(1+r/n)^{nt} = m[(1+r/n)^{nt} - 1]/(r/n)$

17) We know P = $300,000. We need to find A. Use formula $A = P(1+r/n)^{nt}$

18) We know m = $400. We need to find P. Use formula $P(1+r/n)^{nt} = m[(1+r/n)^{nt} - 1]/(r/n)$

19) We know A = $350,000. We need to find P. Use formula $A = P(1+r/n)^{nt}$

20) We know A = $500,000. We need to find m. Use formula $A = m[(1+r/n)^{nt} - 1]/(r/n)$

6.7 1) \$870 = A 2) 81,596 3) \$1190.50 = m
4) \$1755.93 = m 5) P = \$12156.72
6) \$160383.25 7) m = \$289.28
8) P = 16290.63 loan so value = \$19290.63
9) P = \$2085.33 10) P = \$688,675.54
11) \$497897.83
12) \$928.94 if r = .07 or \$1077.95 if r = .05
13) A = \$3447.31
14) a) m = \$1643.90 b) \$128451.61
15) \$9898.48
16) PV = P = \$1213539.16 FV = A = \$5745936.81
17) \$6669.70 = m 18) \$767123287.67
19) m = \$2375.25 20) 109,619
21) \$5805.92 = A 22) P = \$2138.67
23) \$1523.33 24) \$276.68 = m
25) If leasing P = \$7835.35, and the down payment of \$750 brings the present value to \$8585.33
Purchasing for \$8000 is cheaper.
26) Cost in 5 years is \$20615.73
Sinking fund deposit m = \$833.79 / quarter
27) Western Bank r_{EFF} = .0597 or 5.97%
City Bank is better
28) m = \$404.57
29) \$300 per month for 5 years: P = \$14900.82
\$500 per month for 3 years: P = \$16026.59 is better
30) 10.19 years = t
31) \$177692.68 = A
32) \$20000 + \$500/month for 10 years: P = \$61210.74
\$12000 + \$1000/month for 5 years: P = \$61318.43 better
33) P = \$16384.77
34) Monthly payment \$2204.21
Total paid \$793515.60 = (2204.21)(360)
Interest \$368515.6 = 793515.60 - 425000
35) 15.53 years